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Some new results on a linear equation of the second order

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ABSTRACT

Work on solving the second order linear oscillation defined by Eq. (1.1), with continuous and positive coefficient $\Phi(x)$ that satisfies Lipschitz's condition on semi-axis $[0, +\infty)$ and the divergence of $\int_0^{+\infty}(\Phi - G'^2)\,dx$, had started since the 1830s with Sturm's theorems. This paper presents generalizations as well as a simplification of classical Sturm's theorems on the location and the position of zero oscillations, which have not been included in Amrein et al. (2005) [5]. Besides, according to results from Dimitrovski and Mijatović (1997) [1], Dimitrovski et al. (2007) [2] and Dimitrovski et al. (2007) [4], we add some ideas and supplements (Theorems 2.4–2.8, 2.10 and 2.11) to the classical Sturm's theory of oscillations.

1. Introduction and preliminaries

The canonical form of a linear homogeneous differential equation of the second order is

$$y'' + \Phi(x)y = 0 \tag{1.1}$$

under conditions:

1. $\Phi(x)$ satisfies Lipschitz's condition and is a continuous function in area $D = [0, +\infty)$;
2. $\Phi(x) > 0$;
3. $\int_0^{+\infty} \Phi(x)dx$ diverges.

In monographs [1–3], we have presented a new approach to the issues of this equation, using a method we named **series-iteration method** and which not only enables the improvement of Sturm's theorems, but also opens up perspectives for solving new problems.

According to the literature [1,2,4,5], the need for the introduction of the new special function which we named Sturm's functions is established. These will be continuous oscillatory functions with variable amplitudes, periods (that is, distances between zeros), frequencies and pseudo-phases. For the purpose of illustration, the need for their comparison with elementary functions

$$\begin{aligned} y_1 &= F(x)\cos G(x) \\ y_2 &= F(x)\sin G(x) \end{aligned} \tag{1.2}$$

arises, which according to the theorem on the oscillation of the solution to equation (1.1), in an elementary way approximately include the majority of solutions of Eq. (1.1), as well as the connection of coefficients $\Phi(x)$ with amplitude

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